

**PION WAVE FUNCTION FROM QCD SUM RULES  
WITH NONLOCAL CONDENSATES***Talk at the Workshop "Continuous Advances in QCD",  
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Newport News, VA 23606, USA***ABSTRACT**

We investigate a model QCD sum rule for the pion wave function  $\varphi_\pi(x)$  based on the non-diagonal correlator whose perturbative spectral density vanishes and  $\Phi(x, M^2)$ , the theoretical side of the sum rule, consists of condensate contributions only. We study the dependence of  $\Phi(x, M^2)$  on the Borel parameter  $M^2$  and observe that  $\Phi(x, M^2)$  has a humpy form, with the humps becoming more and more pronounced when  $M^2$  increases. We demonstrate that this phenomenon reflects just the oscillatory nature of the higher states wave functions, while the lowest state wave function  $\varphi_\pi(x)$  extracted from our QCD sum rule analysis, has no humps, is rather narrow and its shape is close to the asymptotic form  $\varphi_\pi^{as}(x) = 6x(1-x)$ .

**1. QCD sum rules and pion wave function**

The pion wave function  $\phi_\pi(x)$  is the basic object in the perturbative QCD (pQCD) description of hard exclusive processes involving the pion:  $\phi_\pi(x)$  is the probability amplitude to find the pion in a state composed of its two valence quarks carrying the fractions  $xP$  and  $(1-x)P$  of its large longitudinal momentum  $P$ . More rigorously, the pion wave function  $\phi_\pi(x, \mu)$  can be defined as the function, whose  $N$ -th moment is given by the matrix element of a local operator with  $N$  covariant derivatives<sup>1,2</sup>:

$$\{P^\nu P^{\nu_1} \dots P^{\nu_N}\} \int_0^1 \phi(x; \mu) x^N dx = i^{N-1} \langle 0 | \bar{d} \gamma_5 \{ \gamma^\nu D^{\nu_1} \dots D^{\nu_N} \} u | \pi^+, P \rangle |_\mu \quad (1)$$

where  $\{\dots\}$  denotes the symmetric-traceless part of a tensor and  $\mu$  is the renormalization parameter for the composite operator  $\mathcal{O}_N$ . Instead of  $x$ , it is more convenient sometimes to use the relative variable  $\xi$  defined by  $x \equiv (1 + \xi)/2$ . In the limit of exact  $u$ - $d$  symmetry, the pion wave function is an even function of  $\xi$ , *i.e.*, all odd  $\xi$ -moments of  $\phi(x; \mu)$  vanish. This definition of the wave function implies that its integral normalization is fixed by the matrix element of the axial current:

$$\int_0^1 \phi(x; \mu) x^N dx = f_\pi, \quad (2)$$

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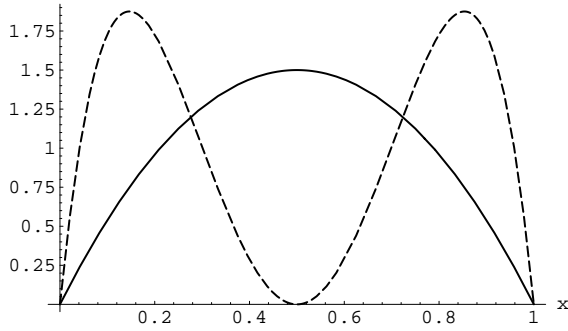


Figure 1: Normalized pion wave function  $\varphi_\pi(x)$ : asymptotic limit (solid line) and CZ-model (dashed line).

where  $f_\pi \approx 133 \text{ MeV}$  is the pion decay constant. In many cases, it is convenient to use the normalized wave function  $\varphi_\pi(x; \mu) \equiv \phi_\pi(x; \mu)/f_\pi$  whose integral is simply 1. The  $\mu$ -dependence of  $\varphi_\pi(x; \mu)$  is governed by the evolution equation which follows from the renormalization-group equation for the composite operators<sup>2</sup>. In the  $\mu \rightarrow \infty$  limit  $\varphi_\pi(x, \mu)$  has a simple and natural form<sup>2</sup> (see also<sup>3</sup>)

$$\varphi_\pi(x; \mu \rightarrow \infty) \equiv \varphi_\pi^{as}(x) = 6x(1-x). \quad (3)$$

However, one is usually interested in the form of  $\varphi_\pi(x; \mu)$  at low values of the renormalization parameter  $\mu \sim 1 \text{ GeV}$  relevant to experimentally accessible situations. This form is determined by non-perturbative QCD dynamics and, in principle, it may strongly differ from the asymptotic limit. To calculate the pion wave function at low values of the probing parameter  $\mu$ , one should take into account non-perturbative aspects of QCD. The closest to pQCD and one of the most popular non-perturbative approaches is provided by QCD sum rules<sup>4</sup> which incorporate information about the non-trivial structure of the QCD vacuum *via* the operator product expansion.

The first application of QCD sum rules to the pion wave function  $\phi_\pi(x)$  was the calculation of the pion decay constant  $f_\pi$ , *i.e.*, the zeroth moment of  $\phi_\pi(x)$ , in the pioneering paper by Shifman, Vainshtein and Zakharov<sup>4</sup> who considered the correlator of two axial currents and calculated  $f_\pi$  within 5% accuracy. Next step was made by Chernyak and A.Zhitnitsky<sup>5</sup> who tried to construct the whole pion wave function  $\varphi_\pi(x)$  using information about its lowest moments calculated from the QCD sum rule analysis for correlators of the axial current with the above mentioned local operators  $\mathcal{O}_N$ . The model wave function chosen by CZ<sup>5</sup>

$$\varphi_\pi^{CZ}(x) = 30x(1-x)(1-2x)^2, \quad (4)$$

has the moments  $\langle \xi^2 \rangle^{CZ} \approx 0.43$  and  $\langle \xi^4 \rangle^{CZ} \approx 0.28$ , to be compared with  $\langle \xi^2 \rangle^{as} = 0.2$  and  $\langle \xi^4 \rangle^{as} = 3/35$  for the asymptotic wave function. The large value of these moments dictates the characteristic double-humped shape of the CZ wave function: it has maxima at  $x \approx 0.15$  and  $x \approx 0.85$  and a zero at the midpoint  $x = 0.5$  (see Fig.1). Such a form for the wave function of a lowest state looks rather strange. Quantum-mechanics-based intuition would rather suggest that the ground state wave function has a shape like that of the asymptotic wave function: without humps, nodes or zeros. One can also expect that all such peculiarities should appear – and in increasing number(!) – for the wave functions of radial excitations.

Formally, the reason for the exotic shape of the CZ model wave function can be traced to the structure of the nonperturbative (condensate) terms in their sum rule. Being written directly for the wave function  $\varphi_\pi(x)$ , it reads:

$$\begin{aligned} f_\pi^2 \varphi_\pi(x) + (\text{higher states}) &= \frac{3M^2}{2\pi^2} x(1-x) + \frac{\alpha_s \langle GG \rangle}{24\pi M^2} [\delta(x) + \delta(1-x)] \\ &+ \frac{8}{81} \frac{\pi \alpha_s \langle \bar{q}q \rangle^2}{M^4} \{11[\delta(x) + \delta(1-x)] + 2[\delta'(x) + \delta'(1-x)]\}. \end{aligned} \quad (5)$$

The perturbative loop contribution in this sum rule has a smooth behavior coinciding with the asymptotic  $x(1-x)$  shape. On the other hand, the condensate terms are strongly peaked at the end-points  $x=0$  and  $x=1$ , *i.e.*, in the regions where one of the quarks has zero momentum. The standard ansatz for the higher states is to model them by perturbative spectral density, which is  $\sim x(1-x)$  in this case. As a result, there remains only one state, the pion, whose wave function has to reflect the presence of the condensate peaks at  $x=0$  and  $x=1$ . In other words, the CZ wave function looks like a compromise between the smooth perturbative loop behavior  $\sim x(1-x)$  and the condensate contributions forcing strong enhancements at  $x=0$  and  $x=1$ .

Earlier<sup>7</sup>, we argued that taking the CZ sum rule (5) at face value amounts to assumption that vacuum quarks have zero momentum, which is an approximation with a limited applicability range. In general, one would expect that vacuum quarks have a smooth distribution in momentum, and, hence, a  $\delta(x)$  term, say, should be treated only as the first term of an expansion of a smooth function in a series over  $\delta(x)$  and its derivatives. If the generating smooth function is not very narrow, then the condensate peaks are not as drastic as the  $\delta(x) + \delta(1-x)$ -approximation, and the impact of the condensate corrections on the pion wave function is much milder<sup>7</sup>.

## 2. Nondiagonal correlator

In what follows, we would like to concentrate on another subtle point of the standard QCD sum rule analysis of hadronic wave functions, namely, on the implicit assumption that higher states can be modeled by (“are dual to”) a perturbative spectral density. In fact, this assumption is in an obvious conflict with a standard quantum-mechanical situation, when the ground state has a monotonous positive-definite wave function, its first radial excitation has one zero, the second has two and higher state wave functions become more and more oscillating. To study this problem in its cleanest form, it makes sense to analyze a sum rule with vanishing perturbative density. This can be easily arranged by taking a non-diagonal correlator<sup>8</sup>, *e.g.*, the correlator of the generic operator  $\mathcal{O}_N$  with the pseudoscalar current  $\bar{d}\gamma_5 u$  rather than with the axial current  $\bar{d}\gamma_5 \gamma_\nu u \equiv \mathcal{O}_0$ . Then, for massless quarks, all the perturbative terms (*i.e.*, those corresponding to the unity operator) of the operator product expansion are zero because an odd number of gamma-matrices would be involved in any trace, and only condensate terms appear on the theoretical side of the sum rule. To the lowest order of  $\alpha_s$ , all these terms have a singular behaviour like  $\delta(x)$ ,  $\delta'(x)$ , *etc.*

The correlator of  $\bar{d}\gamma_5 u$  and  $\bar{u}\gamma_5 \gamma D^N d$  has a remarkably simple structure in the limiting case  $N=0$  both on the theoretical and phenomenological sides of the sum rule:

- a) there is only one particle – the pion – which has nonzero projections on both the axial current  $\bar{d}\gamma_5 \gamma_\mu u$  and the pseudoscalar current  $\bar{d}\gamma_5 u$ ;
- b) for massless quarks, a single operator – the quark condensate – survives in the operator product expansion (a proof can be found in the SVZ paper<sup>4</sup>). Two other, formally allowed terms  $\langle \bar{q}D^2 q \rangle$

and  $\langle \bar{q}(\sigma G)q \rangle$  cancel each other (recall that  $\langle \bar{q}D^2q \rangle = \frac{1}{2}\langle \bar{q}ig(\sigma G)q \rangle$ ). This leads to the well-known PCAC relation

$$\langle 0|\bar{d}\gamma_5 u|\pi \rangle = \frac{i}{f_\pi}(\langle \bar{u}u \rangle + \langle \bar{d}d \rangle). \quad (6)$$

The relevant QCD sum rule, in its borelized form, with  $M^2$  being the Borel parameter characterizing the exponential suppression of the higher states contribution, looks as follows:

$$\langle \xi^N \rangle_\pi + \langle \xi^N \rangle_{\pi'} e^{-m_{\pi'}^2/M^2} + \langle \xi^N \rangle_{\pi''} e^{-m_{\pi''}^2/M^2} + (\text{higher states}) = \frac{1 + (-1)^N}{2} + O(1/M^2), \quad (7)$$

where  $O(1/M^2)$  includes the power suppressed contributions due to higher condensates  $\langle \bar{q}D^2q \rangle$ ,  $\langle \bar{q}(\sigma G)q \rangle$ , *etc.* These contributions vanish for  $N = 0$ , *i.e.*, their coefficients contain factor  $N$ .

Furthermore, since only the pion term survives on the l.h.s. of this sum rule in the specific case  $N = 0$ , we have:

$$\langle \xi^{N=0} \rangle_\pi = 1, \quad \langle \xi^{N=0} \rangle_{\pi'} = \langle \xi^{N=0} \rangle_{\pi''} = \dots = 0. \quad (8)$$

A simple observation is that the (“axial”) wave functions of the higher pseudoscalar mesons  $\pi'$ ,  $\pi''$ ,  $\dots$  must (!) have oscillations to produce zero total integrals. In principle, the pion wave function may have oscillations or humps as well, but this is not mandatory.

As before, we rewrite the sum rule directly for the wave functions:

$$\varphi_\pi(x) + \varphi_{\pi'}(x) e^{-m_{\pi'}^2/M^2} + \dots = \frac{\delta(x) + \delta(1-x)}{2} + a \langle \bar{q}D^2q \rangle \{ \delta'(x) + \delta'(1-x) \} + \dots \quad (9)$$

Note, that higher condensates cannot have  $\delta(x)$  or  $\delta(1-x)$  coefficients, since all higher condensate terms must disappear after one takes the zeroth moment of this sum rule.

Our last comment here is that the smooth functions  $\varphi_\pi(x), \varphi_{\pi'}(x), \dots$  on the l.h.s. of the sum rule can be produced only by an infinite summation of singular distributions  $\delta^n(x), \delta^n(1-x)$  associated with the local condensates.

### 3. Nonlocal condensates

In the coordinate representation, the contribution of the simplest diagram is given by the product of the perturbative propagator  $S(z) \sim (z^\mu \gamma_\mu)/z^4$  and the nonlocal condensate  $\langle \bar{q}(0)q(z) \rangle$ . Next term (evaluated in the Fock-Schwinger gauge, which is the most convenient for the QCD sum rule calculations) is proportional to  $\langle \bar{q}(0)(\sigma G(0))q(z) \rangle (z^\mu \gamma_\mu)/z^2$ . Performing the Taylor expansion of the nonlocal condensates in  $z^2$  produces the OPE in terms of local condensates. The resulting sum rule for the wave function has the structure of an expansion over the delta functions  $\delta(x), \delta(1-x)$  and their derivatives.

Our strategy<sup>7</sup> is to avoid the Taylor expansion to preserve the smoothness properties of the objects involved on the theoretical side of the sum rule, *i.e.*, keep together all terms generated by a particular nonlocal condensate. As the next step, we construct model expressions for the nonlocal condensates to see how the properties of the nonlocal condensates affect the form of the pion wave function extracted from the relevant sum rule.

It is convenient to parametrize the  $z^2$ -dependence of the simplest bilocal quark condensate  $\langle \bar{q}(0)q(z) \rangle \equiv \langle \bar{q}(0)q(0) \rangle Q(z^2)$  with the help of a Laplace-type representation

$$Q(z^2) = \int_0^\infty e^{sz^2/4} f(s) ds. \quad (10)$$

The spectral function  $f(s)$  may be called “the distribution function of quarks in the vacuum” since its  $n$ th moment is proportional to the matrix element of the local operator with  $D^2$  to  $n$ th power:

$$\int_0^\infty s^N f(s) ds \sim \frac{\langle \bar{q}(D^2)^N q \rangle}{\langle \bar{q}q \rangle}. \quad (11)$$

In particular, for the lowest two moments one has

$$\int_0^\infty f(s) ds = 1 \quad (12)$$

and

$$\int_0^\infty s f(s) ds = \frac{1}{2} \frac{\langle \bar{q}(D^2) q \rangle}{\langle \bar{q}q \rangle} \equiv \frac{\lambda_q^2}{2}, \quad (13)$$

with  $\lambda_q^2$  having the meaning of the average virtuality of vacuum quarks.

In a similar way, parametrizing the quark-gluon nonlocal condensate  $\langle \bar{q}(0)ig(\sigma G(0))q(z) \rangle \equiv \langle \bar{q}ig(\sigma G)q \rangle Q_1(z^2)$ , one can introduce the quark-gluon distribution function  $f_1(s)$ . Since

$$\langle \bar{q}D^2 q \rangle = \frac{1}{2} \langle \bar{q}ig(\sigma G)q \rangle, \quad (14)$$

there exist a relation between the zeroth moment of  $f_1(s)$  and the first moment of  $f(s)$ :

$$m_0^2 \equiv \int_0^\infty f_1(s) ds = 4 \int_0^\infty s f(s) ds. \quad (15)$$

The standard QCD sum rule estimate<sup>6</sup> for  $m_0^2$  is  $m_0^2 \simeq 0.8 \text{ GeV}^2$ ; since  $\lambda_q^2 = m_0^2/2$ , one can take  $\lambda_q^2 = 0.4 \text{ GeV}^2$ .

Constructing models of nonlocal condensates, one should satisfy also some other constraints. For instance, if one assumes that the vacuum matrix element  $\langle \bar{q}(D^2)^{N_0} q \rangle$  exists, then  $f(s)$  should vanish faster than  $1/s^{N_0+1}$  as  $s \rightarrow \infty$ . So, if all such matrix elements exist,  $f(s)$  must vanish faster than any power of  $1/s$  for large  $s$ . As a possible choice, one may impose that, at large  $s$ , the function  $f(s)$  behaves like  $f(s) \sim e^{-s^2/\sigma^2}$  (Gaussian fall-off), or  $f(s) \sim e^{-s/\sigma}$  (exponential fall-off), *etc.* The opposite, small- $s$  limit of  $f(s)$  is governed by the large- $|z|$  properties of the function  $Q(z^2)$ , *i.e.*, by its behaviour at large space separations or at large values of the imaginary time variable  $\tau = iz_0$ . The latter case can be easily assessed using the QCD sum rule for the heavy-light meson spectrum in the heavy quark effective theory (HQET). In the HQET, the heavy quark has a trivial propagator  $S_Q(z) \sim \delta^3(\mathbf{z})\theta(z_0)$  and, hence, the time dependence of the correlator of two heavy-light currents is determined by the light quark propagator<sup>9</sup>. At large imaginary time  $\tau$ , the correlator is dominated by the lowest state contribution  $\sim e^{-\tau\bar{\Lambda}}$  where  $\bar{\Lambda} = (M_Q - m_Q)|_{m_Q \rightarrow \infty}$  is the lowest energy level of the mesons in HQET. This means that  $Q(z^2) \sim e^{-|z|\bar{\Lambda}}$  for large Euclidean  $z$  and  $f(s) \sim e^{-\bar{\Lambda}^2/s}$  in the small- $s$  region. Numerically,  $\bar{\Lambda}$  is around  $0.45 \text{ GeV}$ .

Combining, in the simplest way, the  $e^{-\bar{\Lambda}^2/s}$  dependence with, say, the Gaussian fall-off at large  $s$ , we arrive at the ansatz

$$f(s) = N e^{-\bar{\Lambda}^2/s - s^2/\sigma^2} \quad (16)$$

where  $\bar{\Lambda}^2 = 0.2 \text{ GeV}^2$ , the normalization constant  $N$  is fixed by eq.(12) and the  $\sigma$ -parameter is fixed by eq.(13), where for the average virtuality of vacuum quarks we take the usual QCD sum rule value<sup>6</sup>  $\lambda_q^2 \simeq 0.4 \text{ GeV}^2$ .

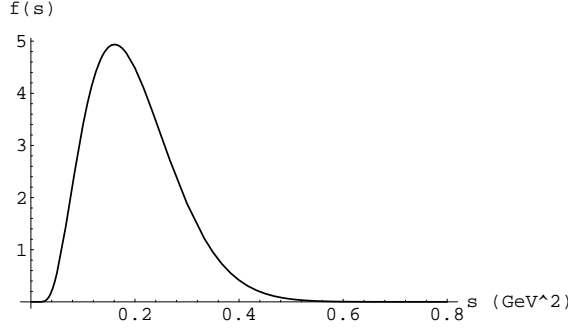


Figure 2: Our model for the vacuum distribution function  $f(s)$ .

#### 4. Sum rule

Now, the sum rule for the pion wave function can be written as

$$\begin{aligned} \varphi_\pi(x) &+ \varphi_{\pi'}(x)e^{-m_{\pi'}^2/M^2} + \varphi_{\pi''}(x)e^{-m_{\pi''}^2/M^2} + \dots \equiv \Phi(x, M^2) \\ &= \left\{ \frac{M^2}{2}(1-x)f(xM^2) + \frac{1}{8}f_1(xM^2) + F(x, M^2) \right\} + (x \rightarrow 1-x) \end{aligned} \quad (17)$$

where  $f, f_1, \dots$  are the vacuum distribution functions corresponding to the lowest nonlocal condensates  $\langle \bar{q}q \rangle$  and  $\langle \bar{q}\sigma Gq \rangle$ , respectively, and the function  $F(x, M^2)$  is given by higher nonlocal condensates  $\langle \bar{q}GGq \rangle, \langle \bar{q}GGGq \rangle, \text{etc.}$

As discussed above, the sum rule reduces to an extremely simple form if one takes its zeroth moment:

$$\int_0^1 \Phi(x, M^2) dx = 1, \quad (18)$$

with only the simplest quark condensate term contributing to “1” in the r.h.s. This imposes the following relations for the two lowest terms of the nonlocal condensate expansion:

$$\begin{aligned} \int_0^{M^2} f(s) ds &= 1, \\ \int_0^{M^2} f_1(s) ds &= 4 \int_0^{M^2} s f(s) ds. \end{aligned} \quad (19)$$

The second relation reflects the cancellation between  $\langle \bar{q}\sigma Gq \rangle$  and  $\langle \bar{q}D^2q \rangle$  terms of the local expansion. For remaining condensates we have

$$\int_0^1 F(x, M^2) dx = 0. \quad (20)$$

By virtue of eqs.(12, 15), the first two relations are satisfied for  $M^2 = \infty$  and since, at large  $s$ , the distribution functions are supposed to vanish faster than any power of  $1/s$ , the violation of the above finite- $M^2$  relations also drops faster than any power of  $1/M^2$  at large  $M^2$ . This is consistent with the fact that contributions decreasing faster than any power may be missed by the operator product

expansion. The practical lesson is that the Borel parameter  $M^2$  should be taken in the region where the violation of the normalization condition (18) is sufficiently small.

To fix the sum rule, one should specify a model for the quark-gluon nonlocal condensate and say something about the higher contributions denoted by  $F(x, M^2)$ . As we will see, our procedure of extracting wave functions from the sum rule is perfectly linear, in the sense that each nonlocal condensate term on the theoretical side of the sum rule produces an additive contribution to all the wave functions on its l.h.s. Hence, one can split each wave function into respective parts generated by *a)* the lowest quark condensate, *b)* quark-gluon condensate, *c)* next condensate, *etc.* Then one can study separately the resulting sum rules, each having only one type of the nonlocal condensate on its r.h.s. Finally, one should add the contributions extracted from each of these partial sum rules. In fact, to illustrate general features of the fitting procedure, it is sufficient to analyze the sum rule containing the lowest nonlocal condensate on its theoretical side. However, since the two lowest nonlocal condensates are related by eq. (19), these two terms should be better considered together.

The structure of the nonlocal quark-gluon condensate is specified by the relevant distribution function  $f_1(s)$ . Information about this function, in principle, can be also obtained from a (future) study of the QCD sum rules in the heavy quark limit. Lacking such information at the moment, we will assume the simplified ansatz that  $f_1(s)$  coincides with the function  $f(s)$  governing the  $z^2$ -dependence of the simplest nonlocal quark condensate. This assumption is not crucial and it does not affect qualitative features of our analysis.

## 5. Fitting sum rule

Now we can write down our model sum rule for the “axial” wave functions of the pseudoscalar mesons:

$$\begin{aligned} \varphi_\pi(x) &+ \varphi_{\pi'}(x)e^{-m_{\pi'}^2/M^2} + \varphi_{\pi''}(x)e^{-m_{\pi''}^2/M^2} + \dots \equiv \Phi(x, M^2) \\ &= \frac{M^2}{2} \left( 1 - x + \frac{\lambda_q^2}{2M^2} \right) f(xM^2), + (x \rightarrow 1 - x) \end{aligned} \quad (21)$$

with the function  $f(s)$  specified in the preceding section. For the  $\pi'$ -meson, we will take the experimental mass  $m_{\pi'}^2 \simeq 1.7 \text{ GeV}^2$ .

It is evident from this sum rule that the function  $\Phi(x, M^2)$ , *i.e.*, the weighted sum of all wave functions is given by two humps, which are moving as  $M^2$  changes. As  $M^2$  increases, the humps become narrower, higher and move towards respective boundary points  $x = 0$  or  $x = 1$ , approaching the  $\delta(x)$  or  $\delta(1 - x)$  form in the  $M^2 \rightarrow \infty$  limit. For  $M^2 = 1 \text{ GeV}^2$ , *e.g.*, the function  $\Phi(x, M^2)$  looks very much like the Chernyak-Zhitnitsky wave function (see Fig.3). However, one should remember that  $\Phi(x, M^2)$  is not just equal to the pion wave function: at large  $M^2$  there might be a large contamination from higher states. Taking larger  $M^2$ , *e.g.*,  $M^2 = 1.2 \text{ GeV}^2$  produces even a wider function, while decreasing  $M^2$  to  $0.8 \text{ GeV}^2$  produces a function with closer and lower humps.

The lower  $M^2$ , the more pronounced is the dominance of the pion in the total sum  $\Phi(x, M^2)$ . However, we cannot take too low  $M^2$ , because the operator product expansion might fail. Since the average virtuality  $\lambda_q^2$  of the vacuum quarks is  $0.4 \text{ GeV}^2$ , it is definitely unreasonable to go below the point  $M^2 = 0.4 \text{ GeV}^2$ , because our “large” probing virtuality  $M^2$  should be larger than  $\lambda_q^2$  – otherwise one should expand the correlator in  $1/\lambda_q^2$  rather than in  $1/M^2$ . Taking  $M^2 = 0.4 \text{ GeV}^2$ ,

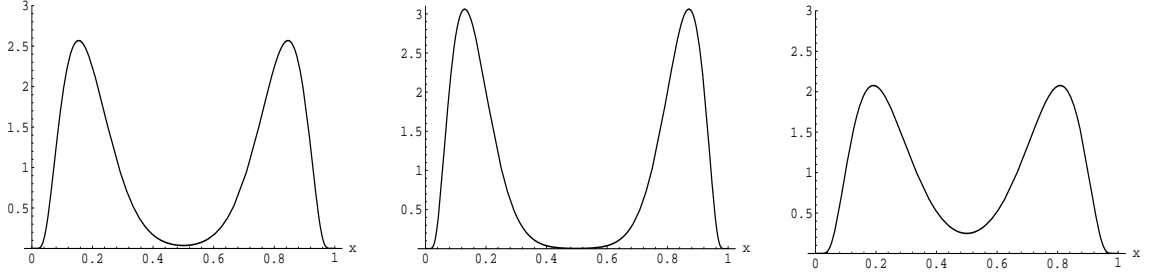


Figure 3: Function  $\Phi(x, M^2)$  for  $M^2 = 1 \text{ GeV}^2$  (left),  $M^2 = 1.2 \text{ GeV}^2$  (middle) and  $M^2 = 0.8 \text{ GeV}^2$  (right).

we observe that  $\Phi(x, M^2)$  is very close to the asymptotic wave function of the pion (see Fig.4). Assuming that the total sum  $\Phi(x, M^2)$  at such low  $M^2$  is completely dominated by the pion, we have to conclude that our model for the nonlocal condensate sum rule suggests that the pion wave function is rather close to its asymptotic form. However, one should be more accurate here, since even the modest increases of  $M^2$  to  $0.5 \text{ GeV}^2$  or  $0.6 \text{ GeV}^2$  induce humps in  $\Phi(x, M^2)$  (see Fig.4).

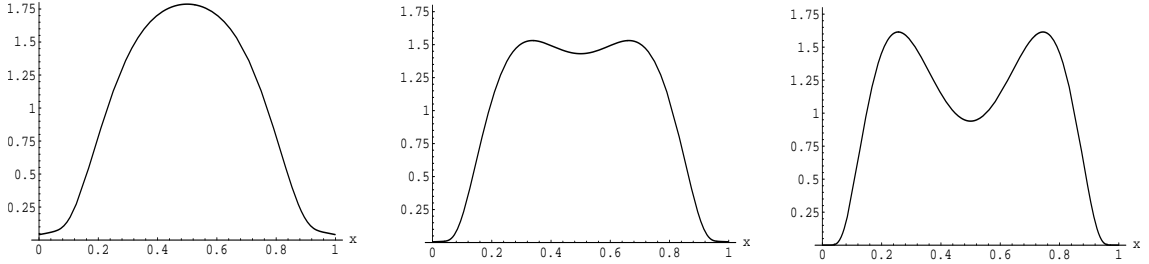


Figure 4: Function  $\Phi(x, M^2)$  for  $M^2 = 0.4 \text{ GeV}^2$  (left),  $M^2 = 0.5 \text{ GeV}^2$  (middle) and  $M^2 = 0.6 \text{ GeV}^2$  (right).

This means that the  $\pi'$ -contribution is visible at  $M^2 \sim 0.5 \text{ GeV}^2$ , and one should better try to fit  $\Phi(x, M^2)$  by two lowest states. Taking two different but close values of  $M^2$ , one can extract the relevant wave functions,

$$\varphi_\pi(x) \simeq \frac{\Phi(x, M_1^2)e^{m_{\pi'}^2/M_1^2} - \Phi(x, M_2^2)e^{m_{\pi'}^2/M_2^2}}{e^{m_{\pi'}^2/M_1^2} - e^{m_{\pi'}^2/M_2^2}}. \quad (22)$$

Choosing  $M_1^2 = 0.5 \text{ GeV}^2$  and  $M_2^2 = 0.55 \text{ GeV}^2$  we obtained the curve for  $\varphi_\pi(x)$  shown in Fig.5 (left). A very close result is obtained if one takes the pair  $M_1^2 = 0.55 \text{ GeV}^2$  and  $M_2^2 = 0.60 \text{ GeV}^2$  (see Fig.5, middle). For definiteness, we will fix the pion wave function as that extracted from the first pair  $M_1^2 = 0.5 \text{ GeV}^2$  and  $M_2^2 = 0.55 \text{ GeV}^2$ . The relevant  $\pi'$  wave function is then shown in Fig.5 (right). Note, that its maxima are by a factor of 10 higher than those of  $\varphi(x)$ . However, this only means that the overall scale characterizing the magnitude of matrix elements  $\langle 0 | \dots | \pi' \rangle$  is essentially larger than  $f_\pi$ . But this is only natural in view of large mass of the  $\pi'$  particle.

To estimate the contribution of the resonances higher than  $\pi'$ , it is convenient to introduce the



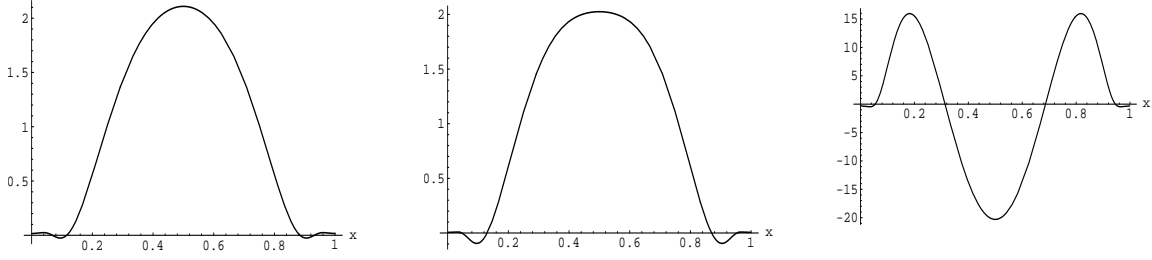


Figure 5: Pion wave function extracted from the two-states fit of  $\Phi(x, M^2)$  performed a) at  $M_1^2 = 0.5 \text{ GeV}^2$  and  $M_2^2 = 0.55 \text{ GeV}^2$  (left) and b) at  $M_1^2 = 0.55 \text{ GeV}^2$  and  $M_2^2 = 0.6 \text{ GeV}^2$  (middle). c) Wave function of the  $\pi'$ -meson extracted from the two-states fit of  $\Phi(x, M^2)$  performed at  $M_1^2 = 0.5 \text{ GeV}^2$  and  $M_2^2 = 0.55 \text{ GeV}^2$  (right).

function

$$\chi_{\pi'}(x, M^2) = (\Phi(x, M^2) - \varphi_{\pi}(x)) e^{m_{\pi'}^2/M^2}. \quad (23)$$

At low  $M^2$ , this function is very close to the  $\pi'$  wave function  $\varphi_{\pi'}(x)$  as determined from our two-states fit. In particular, for the reference points  $M^2 = 0.5 \text{ GeV}^2$  and  $M^2 = 0.55 \text{ GeV}^2$ , this function simply coincides with  $\varphi_{\pi'}(x)$ . For larger  $M^2$ , however, the higher resonances modify its form more and more strongly. This can be seen from Fig.6. It is evident that the difference between  $\chi_{\pi'}(x, M^2)$  and  $\varphi_{\pi'}(x) \equiv \chi_{\pi'}(x, M^2 = 0.55 \text{ GeV}^2)$  increases with  $M^2$ . One can guess that the increase of  $\chi_{\pi'}(x, M^2) - \varphi_{\pi'}(x)$  just reflects the increasing contribution of the next resonance. Looking at the actual curves for the difference  $\chi_{\pi'}(x, M^2) - \varphi_{\pi'}(x)$  at three values,  $M^2 = 0.8 \text{ GeV}^2$ ,  $M^2 = 1 \text{ GeV}^2$  and  $M^2 = 1.2 \text{ GeV}^2$ , (see Fig.6 (right)), one can notice that, to good accuracy, the difference  $\chi_{\pi'}(x, M^2) - \varphi_{\pi'}(x)$  has essentially the same shape for different  $M^2$ , with absolute normalization governed by an  $M^2$ -dependent factor.

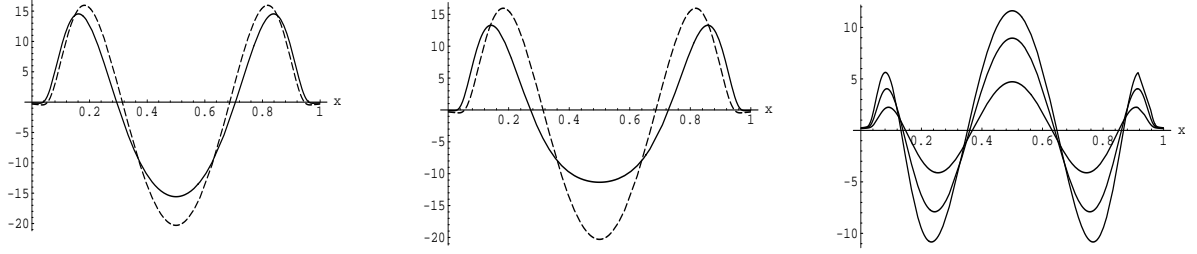


Figure 6: Function  $\chi_{\pi'}(x, M^2)$  for  $M^2 = 0.8 \text{ GeV}^2$  (left), and  $M^2 = 1 \text{ GeV}^2$  (middle), shown together with  $\varphi_{\pi'}(x)$  (dashed line). Right picture shows the increase of the difference  $\chi_{\pi'}(x, M^2) - \varphi_{\pi'}(x)$  when the Borel parameter takes the values  $M^2 = 0.8, 1$  and  $1.2 \text{ GeV}^2$ .

Now, the question is whether one can fit the combination

$$\Phi(x, M^2) - \varphi_{\pi}(x) - \varphi_{\pi'}(x) e^{-m_{\pi'}^2/M^2} \equiv (\chi_{\pi'}(x, M^2) - \varphi_{\pi'}(x)) e^{-m_{\pi'}^2/M^2} \quad (24)$$

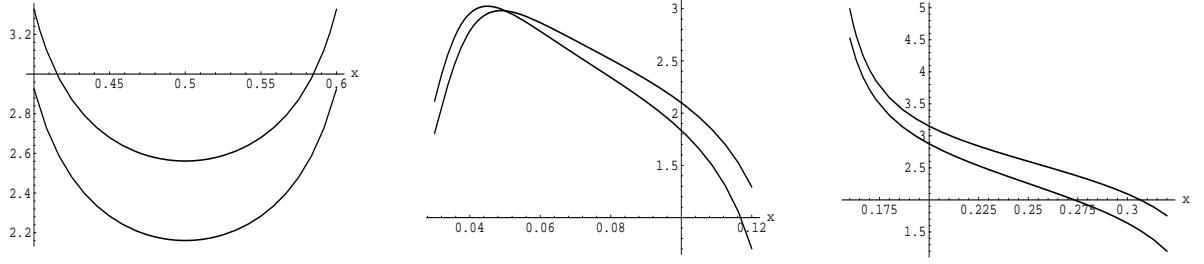


Figure 7: Mass difference  $m_R^2 - m_{\pi'}^2$ , calculated *via* eq.(26) in essential  $x$ -regions.

by the next resonance contribution  $\varphi_R(x)e^{-m_R^2/M^2}$ . This means that we should take the ratio

$$\frac{\chi_{\pi'}(x, M_1^2) - \varphi_{\pi'}(x)}{\chi_{\pi'}(x, M_2^2) - \varphi_{\pi'}(x)} \quad (25)$$

and try to see whether it can be fitted by

$$\frac{e^{m_{\pi'}^2/M_1^2 - m_R^2/M_1^2}}{e^{m_{\pi'}^2/M_2^2 - m_R^2/M_2^2}}.$$

This task can be reformulated as a procedure determining the mass of the third resonance from the relation

$$m_R^2 - m_{\pi'}^2 = \frac{M_1^2 M_2^2}{M_1^2 - M_2^2} \ln \left[ \frac{\chi_{\pi'}(x, M_1^2) - \varphi_{\pi'}(x)}{\chi_{\pi'}(x, M_2^2) - \varphi_{\pi'}(x)} \right]. \quad (26)$$

Again, we take two pairs: *a*)  $(M_1^2, M_2^2) = (1, 0.8) \text{ GeV}^2$  and *b*)  $(M_1^2, M_2^2) = (1.2, 0.8) \text{ GeV}^2$  and plot the r.h.s. of eq.(26) for three  $x$ -regions (see Fig.7). Because of the zeros of  $\chi_{\pi'}(x, M_1^2) - \varphi_{\pi'}(x)$ , the curves are not as constant as one might wish. Still, one can safely state that  $m_R^2 - m_{\pi'}^2 = 2.5 \pm 0.5 \text{ GeV}^2$  is a reasonable estimate. For the mass itself, this gives  $m_R^2 = 4.2 \pm 0.5 \text{ GeV}^2$ , which translates into a rather narrow prediction for the  $\pi''$  mass:  $m_{\pi''} = 2.05 \pm 0.15 \text{ GeV}$ .

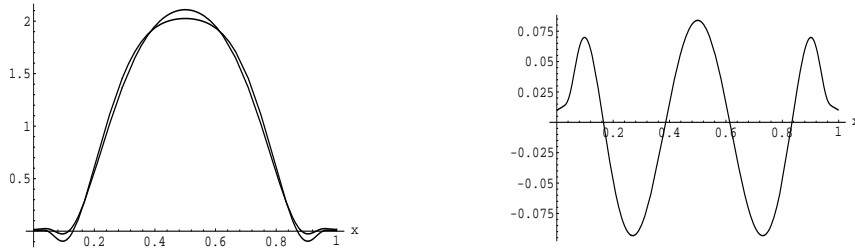


Figure 8: Pion wave functions from Fig.4 (left) and their difference (right).

Though the third resonance is rather massive, it is not completely invisible in the low- $M^2$  region. In particular, looking at the difference between the two pion wave functions shown in Fig. 4 (recall that they were extracted from two-states fits performed for different  $M^2$  pairs), one can see that the resulting curve has the shape specific for the third resonance (Fig.8).

## 6. Summary and conclusions

In this paper, we considered a model sum rule for the pion wave function. A specific feature of this sum rule is that the usual perturbative contribution is absent altogether, and the theoretical side of the sum rule is given by the condensate contributions only. To represent the latter, we incorporated nonlocal condensates. As a result,  $\Phi(x, M^2)$ , the weighted sum of wave functions related to pion and its radial excitations, was given by a curve generated by two humps which were moving to the end points  $x = 0$  and  $x = 1$ , raising in height with increasing Borel parameter  $M^2$ . On the other hand, the relative weight of the higher states increases when  $M^2$  gets larger. This clearly indicates that the peaks observed in  $\Phi(x, M^2)$  for  $M^2 > 0.4 \text{ GeV}^2$  reflect only the oscillatory nature of the wave functions related to the pion excitations. Our explicit fits confirmed this expectation: the pion wave function extracted from this sum rule has no humps and is rather narrow, despite all the humpy nature of  $\Phi(x, M^2)$ . This result has evident implications for the CZ sum rule based on a diagonal correlator: the peaks in the end-point regions generated by the condensates contribution, reflect the oscillatory components in the higher states wave functions rather than the humpy wave function of the ground state, the pion.

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